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SIMULATION OF THE HEAT TRANSFER FROM A LINEAR PULSED-HEAT SOURCE IN THERMOPHYSICAL MEASUREMENTS

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The process of heat transfer from a linear pulsed-heat source in a semibounded body has been investigated. A nondestructive method of determining the thermophysical parameters of a material by the working portion of its thermogram, which allows one to substantially decrease the measurement error, has been developed. Since the model developed is linear by the parameters used, it allows one to estimate, using classical statistical methods, the random error in measuring the thermophysical parameters of materials.

Methods of determining the thermophysical parameters of a material with the use of a pulsed-heat source generating a directed heat flux and, consequently, a nonstationary temperature perturbation in a small region of a sample have considerable technical possibilities since they allow one to independently determine two thermophysical parameters of the material [1, 2]. These methods also call for simple facilities for their technical realization and a short experimental time [3–6]. However, the indicated methods should be further improved since, in them, the thermophysical parameters of a material are determined on the basis of the data of indirect experiments with the use of definite mathematical models. Because of this, the exactness and reliability of the thermophysical parameters of a material determined with the use of these models depend, by and large, on the correctness of the mathematical description of the thermal processes occurring in the material in the process of measurements. In the present work, we considered the theoretical basis of the method of nondestructive control of the thermophysical parameters of materials with the use of a model of nonstationary heat transfer from a linear pulsed-heat source operating on a heat-insulated surface of a semibounded body. In an experiment, the temperature of a material is measured at a point offset by a certain distance from the heat source.

The temperature field formed by a single heat pulse in the material of a semibounded body (Fig. 1) is defined [5, 6] as

$$T(r, \tau) = \frac{Q}{2\pi\lambda\tau} \exp\left[-\frac{r^2}{4a\tau}\right]. \quad (1)$$

If a sequence of $n + 1$ heat pulses is supplied with a period $\Delta\tau$, the temperature field in the sample is determined from the equation

$$T_{\text{p}}(r, \tau) = \frac{Q}{2\pi\lambda} \sum_{i=1}^{n} \frac{\exp\left[-\frac{r^2}{4a(\tau - (i-1)\Delta\tau)}\right]}{\tau - (i-1)\Delta\tau}, \quad (n-1)\Delta\tau \le \tau \le n\Delta\tau, \quad (2)$$

that is little suitable for calculating its thermophysical parameters.

If the power of heating q of a semibounded medium is constant, the temperature field in it is defined [5] as

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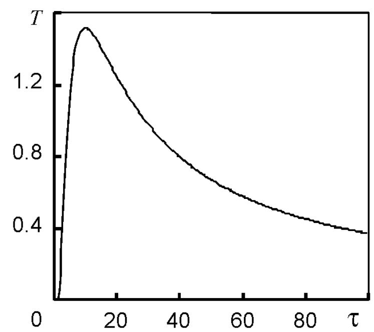


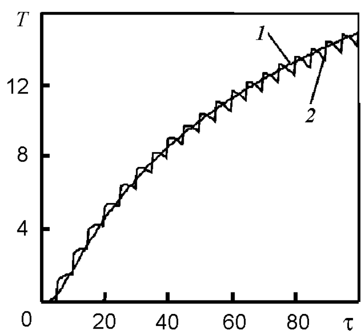


Fig. 1. Dependence $T(\tau)$ in the case of action by a single pulse.

Fig. 2. Dependence $T(\tau)$ in the case where condition (4) is fulfilled and a heat source of constant power (curve 1) or a pulsed-heat source (curve 2) act.

$$T_s(r,\tau) = \frac{q}{2\pi\lambda} \int\limits_{r^2/(4a\tau)}^{\infty} \frac{\exp[-u]}{u}\, du = \frac{q}{2\pi\lambda} \int\limits_{0}^{4a\tau/r^2} \frac{\exp[-1/u]}{u}\, du\,. \tag{3}$$

On condition that

$$q = \frac{Q}{\Delta\tau} \tag{4}$$

it may be assumed (at large τ) that a pulsed-heat source has a constant power (Fig. 2).

In practice, the action of a heat source is not instantaneous. A heater with a power q_0 per unit of its length acts for a time τ_0 (Fig. 3).

The power of a heater is a periodic time function, i.e.,

$$q(\tau) = \begin{cases} q_0\,, & 0 \le \tau \le \tau_0\,, \\ 0\,, & \tau_0 < \tau < \Delta\tau\,, \end{cases} \quad q(\tau + n\Delta\tau) = q(\tau)\,. \tag{5}$$

In this case, condition (4) has the form

$$q = \frac{q_0\tau_0}{\Delta\tau}\,. \tag{6}$$

Using the source method [5, 6], we will write a formula for determining the temperature field formed in a semispace by a linear heat source supplied with power by an arbitrary law:

$$T(r,\tau) = \frac{1}{2\pi\lambda} \int\limits_0^{\tau} \frac{q(u)\exp\left[-\frac{r^2}{4a(\tau-u)}\right]}{\tau - u}\, du\,. \tag{7}$$

The dependence $q(\tau)$ determined from (5) is a periodic function with a period $\Delta\tau$ that can be expanded into the Fourier series:

$$q(\tau) = \frac{q_0\tau_0}{\Delta\tau} + \frac{q_0\tau_0}{\Delta\tau\pi} \sum_{k=1}^{\infty} \frac{1}{k} \left\{ \sin\left[\frac{2k\pi\tau_0}{\Delta\tau}\right] \cos\left[\frac{2k\pi\tau}{\Delta\tau}\right] + \left(1 - \cos\left[\frac{2k\pi\tau_0}{\Delta\tau}\right]\right) \sin\left[\frac{2k\pi\tau}{\Delta\tau}\right] \right\}. \tag{8}$$

Substituting (8) into formula (7), we obtain the following relation:

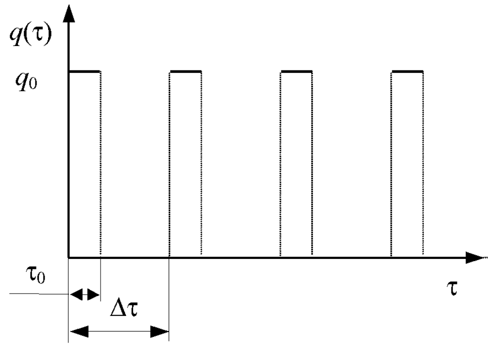


Fig. 3. Power of a pulsed-heat source.

$$T_{\text{p}}(r,\tau)=\frac{q_0\tau_0}{2\pi\lambda\Delta\tau}\int\limits_0^{4a\tau/r^2}\frac{\exp[-1/u]}{u}\,du+$$

$$+\frac{q_0}{2\pi^2\lambda}\sum_{k=1}^{\infty}\frac{1}{k}\left\{\int\limits_0^{\tau}\frac{\left(\sin\left[\frac{2k\pi}{\Delta\tau}(\tau_0-u)\right]+\sin\left[\frac{2k\pi u}{\Delta\tau}\right]\right)\exp\left[-\frac{r^2}{4a(\tau-u)}\right]}{\tau-u}\,du\right\}. \tag{9}$$

From (9), we can obtain an expression, identical to (2), for the temperature field formed by an instantaneous pulsed-heat source.

Decreasing the pulse duration τ_0 and simultaneously increasing the power q_0 so that $q_0\tau_0 = Q$ and taking into account the fact that, in the limit,

$$\lim_{\tau_0\to 0}\left\{\frac{\sin\left[\frac{2k\pi\tau_0}{\Delta\tau}\right]}{\tau_0}\right\}\to\frac{2k\pi}{\Delta\tau},\quad \lim_{\tau_0\to 0}\left\{\frac{1-\cos\left[\frac{2k\pi\tau_0}{\Delta\tau}\right]}{\tau_0}\right\}\to 0,$$

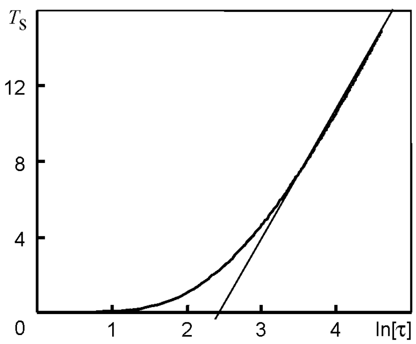
we obtain

$$T_{\text{p}}(r,\tau)=\frac{Q}{2\pi\lambda\Delta\tau}\int\limits_0^{4a\tau/r^2}\frac{\exp[-1/u]}{u}\,du+\frac{Q}{\pi\lambda\Delta\tau}\sum_{k=1}^{\infty}\int\limits_0^{\tau}\frac{\cos\left[\frac{2k\pi u}{\Delta\tau}\right]\exp\left[-\frac{r^2}{4a(\tau-u)}\right]}{\tau-u}\,du. \tag{10}$$

The sums in the right side of expressions (9) and (10) represent bounded and periodic functions of τ [7–10]. At large τ $\left(\int\limits_0^{4a\tau/r^2}\frac{\exp[-1/u]}{u}\,du\right.$ is a monotonically increasing function; as $\tau\to\infty$, $\left.\int\limits_0^{4a\tau/r^2}\frac{\exp[-1/u]}{u}\,du\to\infty\right)$ these sums can be disregarded, i.e., since $q = q_0\tau_0/\Delta\tau$ and $q = Q/\Delta\tau$, it may be assumed that

$$T_{\text{p}}(r,\tau)\approx T_{\text{s}}(r,\tau)=\frac{q}{2\pi\lambda}\int\limits_{r^2/4a\tau}^{\infty}\frac{\exp[-u]}{u}\,du.$$

Let us analyze Eq. (3). It is known [7] that



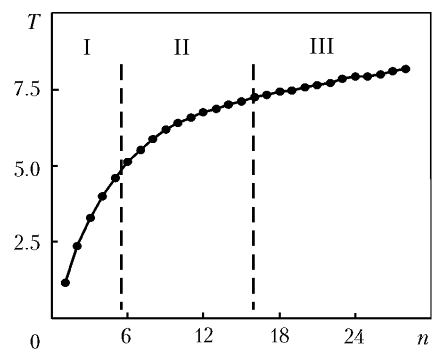


Fig. 4. Dependence $T_s = f(\ln [\tau])$.

Fig. 5. Portions of the thermogram measured for the Ripor-type polyurethane foam.

$$-\int_x^\infty \frac{\exp[-u]}{u}\,du = \ln[x] + \gamma + \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{k\cdot k!}.$$

Using this expression, we write

$$T_s(r,\tau) = \frac{q}{2\pi\lambda}\left(\ln\left[\frac{4a\tau}{r^2}\right] - \gamma - \sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{r^2}{4a\tau}\right)^k}{k\cdot k!}\right) \quad (11)$$

and, at large τ (Fig. 4),

$$T_s(r,\tau) \approx \frac{q}{2\pi\lambda}\left(\ln\left[\frac{4a\tau}{r^2}\right] - \gamma\right) = \frac{q}{2\pi\lambda}\left(\ln[\tau] + \ln[a] - \ln\left[\frac{r^2}{4}\right] - \gamma\right). \quad (12)$$

The dynamics of a thermal process will be characterized by the input action (law of power supply to a heater), the output variable $T(\tau)$, and the system-state variable (as which the heat flux passing through the measurement point can be used). In the general case, three portions can be separated on a thermogram (Fig. 5): I) the heat flux passing through the measurement point changes with time depending on the initial stage of the thermal process; II) the heat fluxes are controlled and are made practically constant; relation (12) is true here, and it will be true also in the case where measurements are performed with account for the actual sizes and heat capacities of the heater and the temperature detectors; III) the sample studied becomes bounded and the heat flux passing through the measurement point becomes variable.

Thus, expression (12) can be used for calculating (working) portion II of the thermogram (Figs. 5 and 6). Since the temperature is measured within certain time intervals $\Delta\tau$, i.e., $\tau = n\Delta\tau$, where $n = 1, 2, 3, \ldots$, expression (12) can be written in the form

$$T(t_{\mathrm{lin}}) = \frac{q}{2\pi\lambda}\left(t_{\mathrm{lin}} + \ln[a] - \ln\left[\frac{r^2}{4\Delta\tau}\right] - \gamma\right). \quad (13)$$

The main expression for calculating the thermophysical parameters of a material is expression (13). To use this expression, it is necessary to know the regime (q, $\Delta\tau$) and design (r) features of the apparatus used for measurements. Therefore, these quantities can be taken as constants of the apparatus under definite experimental conditions. Their values are determined from the calibration experiments (it will suffice to have one sample with known thermophysical parameters).

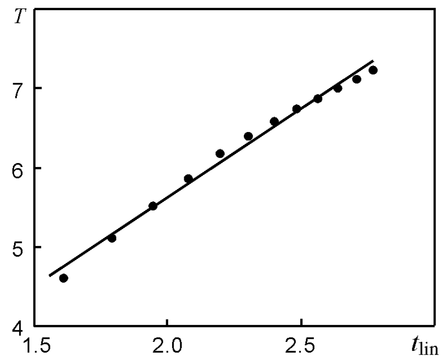


Fig. 6. Second portion of the thermogram.

For the purpose of calculating the thermophysical properties of a material by experimental data, we write expression (13) in the following form:

$$T\,(t_{lin}) = b_1 t_{lin} + b_0\,, \qquad (14)$$

where $b_1 = \dfrac{\alpha}{\lambda}$ and $b_0 = \dfrac{\alpha}{\lambda}\,(\ln|a| - \beta)$ are parameters of the model describing the working portion of the thermogram and $\alpha = \dfrac{q}{2\pi}$ and $\beta = \ln\left[\dfrac{r^2}{4\Delta\tau}\right] + \gamma$ are constants of the apparatus used that are determined by its design features and the experimental conditions.

The thermophysical parameters of the material and the apparatus constants are calculated using the expressions

$$\alpha = \lambda_{st} b_{1st}\,, \qquad (15)$$

$$\beta = \ln\,[a_{st}] - \frac{b_{0st}}{b_{1st}}\,, \qquad (16)$$

$$\lambda = \frac{\alpha}{b_1}\,, \qquad (17)$$

$$a = \exp\left[\frac{b_0}{b_1} + \beta\right]. \qquad (18)$$

Thus, we have proposed a method for determining the boundaries of thermogram portions I–III and the coefficients of Eq. (14) [11].

The model developed with account for different regimes of operation of a measuring system allows one to determine a complex of thermophysical properties of a material — heat conduction and thermal diffusivity. In the case where the thermophysical parameters of a material are determined on the basis of experimental data with the use of dependence (13), the systematic measurement error can be substantially decreased. Since thermophysical parameters of a material are determined by a portion of its thermogram and not by individual points, the influence of the random measurement errors is small. This is especially important for composite materials. In the case of measurement of such a material, the error caused by the difference between the local and mean values of its thermophysical parameters is added to the random error. Since model (13) is linear by its parameters, the random error in measuring the thermophysical parameters of a material in an individual experiment can be estimated using classical statistical methods.

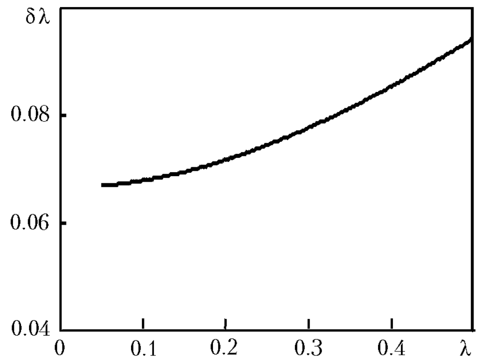


Fig. 7. Dependence $\delta\lambda = f(\lambda)$ calculated by formula (23).

In the method proposed for control of the thermophysical parameters of a material, the random error in measuring the heat conduction and thermal diffusivity is determined, in accordance with the computational procedure described in [11], by the following equations:

$$\delta\lambda = \sqrt{\delta^2\alpha + \delta^2 b_1}\,, \tag{19}$$

$$\delta a = \sqrt{(\delta^2 b_0 + \delta^2 b_1)\left(\frac{b_0}{b_1}\right)^2 + \Delta^2\beta}\,, \tag{20}$$

$$\delta\alpha = \sqrt{\delta^2\lambda_{st} + \delta^2 b_{1st}}\,, \tag{21}$$

$$\Delta\beta = \sqrt{\delta^2 a_{st} + (\delta^2 b_{0st} + \delta^2 b_{1st})\left(\frac{b_{0st}}{b_{1st}}\right)^2}\,. \tag{22}$$

According to the computational procedure used, we first found the differentials of the left and right sides of Eqs. (15)–(18). Then we made substitutions used in the theory of errors: $d\lambda \approx \Delta\lambda$, $d\alpha \approx \Delta\alpha$, $db_1 \approx \Delta b_1$, $da \approx \Delta a$, $db_0 \approx \Delta b_0$, $db_{0st} \approx \Delta b_{0st}$, $db_{1st} \approx \Delta b_{1st}$, and $da_{st} \approx \Delta a_{st}$, where $\Delta\lambda$, $\Delta\alpha$, Δb_1, Δa, Δb_0, Δb_{0st}, Δb_{1st}, and Δa_{st} are the absolute errors in determining λ, α, b_1, a, b_0, b_{0st}, b_{1st}, and a_{st}. The relative errors are determined with account for these substitutions: $\delta\alpha = \Delta\alpha/\alpha$, $\delta b_1 = \Delta b_1/b_1$, etc.

Let us now analyze dependences (19)–(22), obtained from formulas (15)–(18) with the use of the above-described computational procedure, for the purpose of determining the range of measurement of the thermophysical parameters of a material and ways of widening it. It should be noted first of all that the error in calculating the apparatus constants is determined by the error in determining the thermophysical parameters of a standard. The absolute error in determining the coefficients b_0 and b_1 can be assumed to be constant in the first approximation since it will be determined by the error in measuring the temperature (which can be assumed to be constant from experiment to experiment). Then, from expression (19) we find

$$\delta\lambda = \sqrt{\delta^2\alpha + \frac{\Delta^2 b_1 \lambda^2}{\alpha^2}}\,. \tag{23}$$

As is seen from Eq. (23), the relative error $\delta\lambda$ depends on λ (Fig. 7) and increases with increase in the heat conduction of the material. It also follows from expression (23) that this error can be decreased by increasing the coefficient

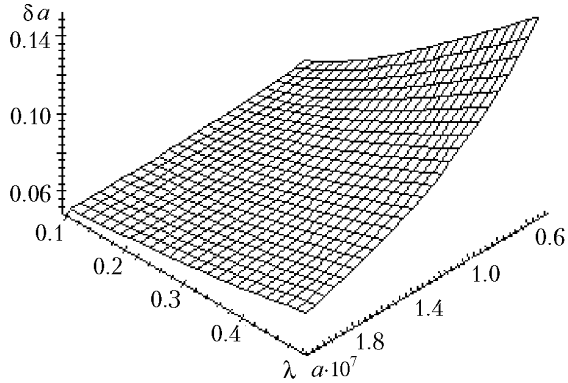


Fig. 8. Dependence $\delta a = f(\lambda, a)$ calculated by formula (24).

α. This can be done by increasing the amount of heat released by a heater, the pulse duration, and the heater power ($\alpha \sim q$).

Let us consider formula (20). From it we obtain, taking into account (17) and (18), that

$$\delta a = \sqrt{(\Delta^2 b_0 + \Delta^2 b_1 (\ln (a) - \beta)^2) \frac{\lambda^2}{\alpha^2} + \Delta^2 \beta} \ . \tag{24}$$

It is seen from expression (24) that the relative error in determining the thermal diffusivity δa is dependent on a and λ of the sample (Fig. 8); in this case, the heat-conduction coefficient λ is a determining factor, an increase in which leads to an increase in the relative error in measuring the thermal diffusivity δa. It follows from Eq. (24) that the relative error in measuring a can be decreased by increasing the apparatus constant α. Using expressions (23) and (24), one can estimate the error in determining the thermophysical parameters of a material by the range of their change. Figures 7 and 8 present results of such an estimation.

Along with the random error in determining the thermophysical parameters of a material, of interest is the systematic error caused by the inaccuracy of the mathematical model. We now consider the influence of the main systematic errors on the accuracy of determining the thermophysical parameters of the materials studied by the method developed.

Systematic errors are caused first of all by the following factors: (a) an actual heater has finite dimensions and a heat capacity, (b) between the heater and the sample as well as between the sample and the temperature detectors there are thermal resistances, (c) a portion of the heat released by the heater is expended in heating the material of a probe.

To determine the influence of the finiteness of the heater dimensions on the accuracy of determining the thermophysical properties of a material, we will consider (1) a heater shaped as a cylinder of radius R, through whose surface the heat flux $\overline{q}_0$ is supplied to the material, (Fig. 9I) and (2) a heater having the form of a strip of width $2h$, through which the heat flux $\overline{q}_0$ is supplied to the material (Fig. 9II).

For the heater of finite dimensions shaped as a cylinder with a radius R, through whose surface the heat flux $\overline{q}_0$ is supplied to the material (Fig. 9I), the problem is solved in the general case [5] in the following way:

$$T(r, \tau) = -\frac{2\overline{q}_0}{\pi\lambda} \int_0^\infty \left(1 - \exp\left[-au^2\tau\right]\right) \frac{J_0(ur)\, Y_1(uR) - Y_0(ur)\, J_1(uR)}{u^2 \left[J_0^2(uR) + Y_1^2(uR)\right]} du \ . \tag{25}$$

At large τ, this expression becomes simpler and has the form [4]

$$T(r, \tau) = \frac{\overline{q}_0 R}{2\lambda} \left(\ln\left(\frac{4a\tau}{r^2}\right) - \gamma\right), \tag{26}$$

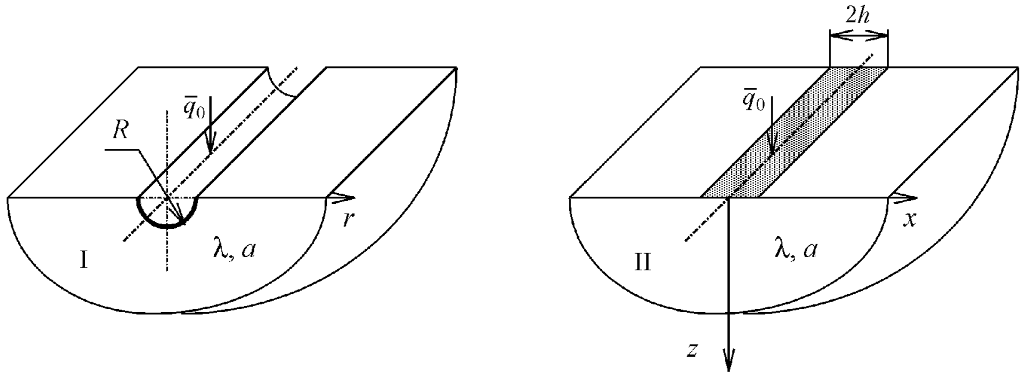


Fig. 9. Diagrams of heaters in the form of a cylinder (I) and a strip (II).

Rewriting (26) in terms of the power per unit heater length ($q_0 = \bar{q}_0 \pi R$, see Fig. 9I), we obtain expression (13).

We now consider a heater of finite dimensions having the form of a strip, through which the heat flux $\bar{q}_0$ is supplied to the material (Fig. 9II). The temperature T at the point with coordinates (x, 0) on the surface of the material studied at the instant of time τ will be determined from the expression [5]

$$T(x, 0, \tau) = \frac{\bar{q}_0 h \mathrm{Fo}^{0.5}}{\pi^{0.5}\lambda} \left\{ \mathrm{erf}\left[\frac{h+x}{2h\mathrm{Fo}^{0.5}}\right] + \mathrm{erf}\left[\frac{h-x}{2h\mathrm{Fo}^{0.5}}\right] - \right.$$

$$\left. - \frac{h+x}{2h(\pi\mathrm{Fo})^{0.5}} \mathrm{Ei}\left[-\frac{(h+x)^2}{4h^2\mathrm{Fo}}\right] - \frac{h-x}{2h(\pi\mathrm{Fo})^{0.5}} \mathrm{Ei}\left[-\frac{(h-x)^2}{4h^2\mathrm{Fo}}\right] \right\}. \tag{27}$$

Using the known relations [7, 8]

$$\mathrm{erf}[x] = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n!(2n+1)}, \tag{28}$$

$$\mathrm{Ei}[-x] = \gamma + \ln(x) + \sum_{n=1}^{\infty} \frac{(-x)^n}{n \cdot n!}, \tag{29}$$

we obtain an expression for the surface temperature $z = 0$ at the point with an x coordinate at large τ:

$$T(x, 0, \tau) = \frac{\bar{q}_0 h}{\lambda\pi} \left\{ \ln(4a\tau) - \frac{h+x}{2h} \ln(h+x)^2 - \frac{h-x}{2h} \ln(h-x)^2 + 2 - \gamma \right\}. \tag{30}$$

Rewriting (30) in terms of the power per unit heater length ($q_0 = \bar{q}_0 2h$, Fig. 9), we obtain

$$T(x, 0, \tau) = \frac{q_0}{2\pi\lambda} \left\{ \ln(4a\tau) - \frac{h+x}{2h} \ln(h+x)^2 - \frac{h-x}{2h} \ln(h-x)^2 + 2 - \gamma \right\}. \tag{31}$$

A more comprehensive analysis of expressions (13) and (31) has shown that they are different. Note also that

$$\lim_{h \to 0} \left\{ \frac{h+x}{2h} \ln(h+x)^2 - \frac{h-x}{2h} \ln(h-x)^2 \right\} =$$

$$= \lim_{h \to 0} \left\{ \frac{h}{2h} \ln\left[(h+x)^2 (h-x)^2\right] + \frac{x}{2h} \ln\left[\frac{(h+x)^2}{(h-x)^2}\right]^2 \right\} = \ln(x^2) - 2.$$

Therefore, on condition that $x >> h$, expression (31) takes the form of formula (13), which can be represented as

$$T(r, \tau) = \frac{q_0}{2\pi\lambda} (\ln (4a\tau) - \ln (r^2) - \gamma) . \tag{32}$$

In this case, (31) and (32) can be written in the unique form

$$T(r, \tau) = T(x, 0, \tau) = \frac{q_0}{2\pi\lambda} [\ln (4a\tau) - C (r \text{ (or } x))] , \tag{33}$$

where $C(r$ (or x)) is a quantity independent of time and the thermophysical parameters of the material studied; it is completely determined by the design features of the probe and represents, in essence, an apparatus constant.

In the method developed, the apparatus constants are determined from calibration experiments carried out with samples whose thermophysical properties are known. In this case, a sample studied is calibrated and its thermophysical parameters are determined by the portion of the thermogram where the temperature-time dependence of the form of (13) is fulfilled.

The foregoing allows us to suggest that the systematic error caused by the finiteness of the heater dimensions does not influence the accuracy of determining the thermophysical properties of a material. As was mentioned above, the systematic error in measuring the thermophysical parameters of a material can be partially estimated in the case where these parameters are determined on the basis of the data of calibration experiments (see (31)–(33)).

Since an actual heater has a heat capacity, a portion of the heat released by it will be expended in heating a sample, i.e., the power supplied to the sample will be equal to $q_0 - q_0'$ and not to q_0. Let us assume that a heater in the form of an infinite cylinder of radius R represents a perfect conductor. In this case, q_0' will be equal to

$$q_0' = C_{\mathrm{h}} \frac{\partial T(R, \tau)}{\partial \tau} . \tag{34}$$

In the first approximation, $T(R, \tau)$ can be determined from expression (13) at $r = R$. The change in the temperature at the point with an r coordinate is determined from the relation

$$T(r, \tau) = \frac{q_0 - q_0'}{2\pi\lambda} \left(\ln \left[\frac{4a\tau}{r^2} \right] - \gamma \right) = \frac{q_0 - C_{\mathrm{h}} \dfrac{q_0}{2\pi\lambda\tau}}{2\pi\lambda} \left(\ln \left[\frac{4a\tau}{r^2} \right] - \gamma \right) \tag{35}$$

or

$$T(r, \tau) = \frac{q_0}{2\pi\lambda} \left(\ln \left[\frac{4a\tau}{r^2} \right] - \gamma \right) - \frac{C_{\mathrm{h}} q_0}{4\pi^2\lambda^2} \left(\frac{\ln [\tau]}{\tau} + \frac{\ln \left[\dfrac{4a}{r^2} \right] - \gamma}{\tau} \right). \tag{36}$$

It is seen from formula (36) that the influence of the heat capacity of the heater will decrease with time since $\lim_{\tau \to \infty} \left(\frac{\ln [\tau]}{\tau} \right) \to 0$ and $\lim_{\tau \to \infty} \left(\frac{\ln \left[\frac{4a}{r^2} \right] - \gamma}{\tau} \right) \to 0$. In this case, the heat capacity of the heater can be disregarded (see (35)) on condition that

$$\frac{C_{\mathrm{h}}}{2\pi\lambda\tau} << 1 . \tag{37}$$

It is seen from expressions (35) and (36) that, in the case where the heat capacity of the heater significantly influences the thermal process, the time dependence of the temperature at the measurement point (thermogram) will differ from expressions (13) and (33). Only the thermogram points at which these functional dependences are fulfilled are taken into account in the method proposed. This allows the conclusion that the heat capacity of the heater in the computational region of the thermogram does not influence the accuracy of determining the thermophysical properties of a material. This can be analogously demonstrated for a heater in the form of a strip and for heat detectors.

Thus, the finiteness of the heater dimensions does not influence the accuracy of determining the thermophysical properties of a material in the case where the apparatus constants are determined by the data of calibration experiments. The heat capacity of the heater in the computational region of the thermogram also does not influence the accuracy of determining the thermophysical properties of a material.

NOTATION

a and a_{st}, thermal diffusivity of the material studied and the standard, m^2/sec; b_0, b_1, and b_{0st}, b_{1st}, coefficients determined by the thermograms measured for the material studied and the standard; C_h, heat capacity of the heater unit length, J/(kg·K·m); Fo, Fourier number; J_0, J_1, Y_0, Y_1, Bessel functions; $2h$, width of the strip, m; n, number of a thermogram point; Q, quantity of heat released per unit heater length, J/m; q, q_0, power per unit heater length, W/m; q_0', power expended in heating the heater, W/m; $\bar{q}_0$, heat-flux density, W/m^2; R, radius of the heater, m; r, x, z, coordinates, m; T, excess temperature, ^{o}C; T_p, temperature in the case of pulsed heating, ^{o}C; T_s, temperature in the case of heating with a constant power, ^{o}C; t_{lin} = ln [n]; i = 1, 2, 3, ..., n; k = 1, 2, 3, ...; u, integration parameter; α, β, C, constants of the apparatus determined by its design features and experimental conditions; $\gamma \approx 0.5772$, Euler number; δ, relative error; Δ, absolute error; $\Delta\tau$, time period, sec; τ, time, sec; τ_0, pulse duration, sec; λ and λ_{st}, heat-conductivity coefficients of the material studied and the standard, W/(m·K). Subscripts: p, pulse; lin, linear; h, heater; st, standard; s, stationary.

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